Journal of Applied Mechanics and Technical Physics, Vol. 49, No. 1, pp. 18–22, 2008

LOSS OF ENERGY DURING THE MOTION OF A VORTEX RING

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UDC 532.527, 555.55

Experimental estimates of energy and energy dissipation of a vortex ring are presented. The energy losses during the motion of a vortex ring and a streamlined solid are compared.

Key words: *vortex ring, energy dissipation, drag of streamlined solids.*

Lavrent'ev and Shabat [1] put forward an idea that it is possible to reduce the drag of solids in a viscous fluid by organizing a flow similar to a flow in a vortex ring. This idea is validated by the following simple experiments. If a mass of air contained in a child's balloon is ejected with an initial velocity of 5–10 m/sec, the balloon will move to a distance equal to 1.5–2.0 m; if the same mass of air is ejected from a circular orifice with the radius equal to the balloon radius, a vortex ring is formed, which moves to a distance greater by a factor of 10 to 15. An impression is created that the motion of the fluid mass in the form of a vortex ring occurs with a substantially lower energy dissipation.

This problem attracted the attention of many researchers. Lugovtsov and Lugovtsov [2] theoretically determined the energy dissipation in a flow with two hollow straight-line vortices of the opposite signs, and Sennitskii [3] measured the drag of two parallel solid cylinders rotating in the opposite directions. The drag of the rotating cylinders was found to be lower than that of non-rotating cylinders, but not to the extent estimated theoretically. Each of these flows is a plane analog of a vortex ring. It seems of interest to directly determine the energy dissipation during the motion of a vortex ring and to compare it with the power necessary for a streamlined solid to move in a viscous medium.

The vortex ring is a toroidal volume of a swirled fluid, which moves in the ambient medium perpendicular to the ring plane with an approximately constant velocity [4]. The fluid motion is axisymmetric, and the vorticity vector (velocity rotor) in the torus is directed along the circumferences coaxial with the circumferential axis of the torus. The toroidal vortex ring captures a certain volume of the fluid around the ring, which moves together with the vortex ring and has the shape similar to an ellipsoid of revolution flattened in the direction of ring motion. This closed volume of the fluid is called the vortex atmosphere. Inside the vortex atmosphere, the fluid circulates along closed streamlines around the vortex core. The motion of the medium surrounding the vortex atmosphere is similar to a non-separated flow around a corresponding solid. Based on the experimentally measured velocity field and velocity of ring motion, one can determine the energy characteristics of the vortex ring.

The field of velocities and the hydrodynamic structure of a real air vortex ring formed by exhaustion of a finite-length ambient jet from a circular nozzle were obtained in [5] for the first time. The axisymmetric velocity field of a laminar vortex ring was measured by two hot-wire probes mounted on the vortex path at a certain distance from the nozzle-exit cross section, where the vortex structure was considered to be completely formed. The translational velocity of the vortex ring was determined by photographing the vortex motion as a function of time. The vortex ring was simultaneously visualized with the use of smoke supplied into the vortex-generator nozzle. The vortex motion was recorded through a narrow slot diaphragm aligned parallel to the vortex motion onto a film moving uniformly perpendicular to the slot. In such a manner, the film shows the law of the vortex motion in time, which allows its translational velocity to be determined. One picture is shown in Fig. 1 (t_0 is the

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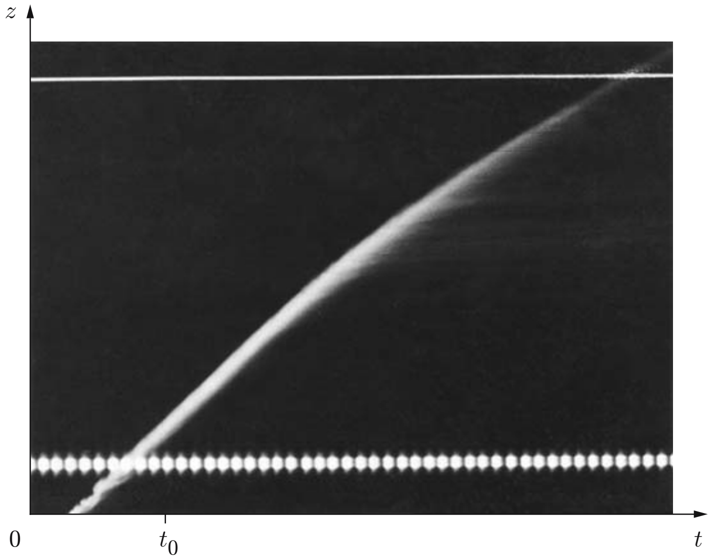

Fig. 1

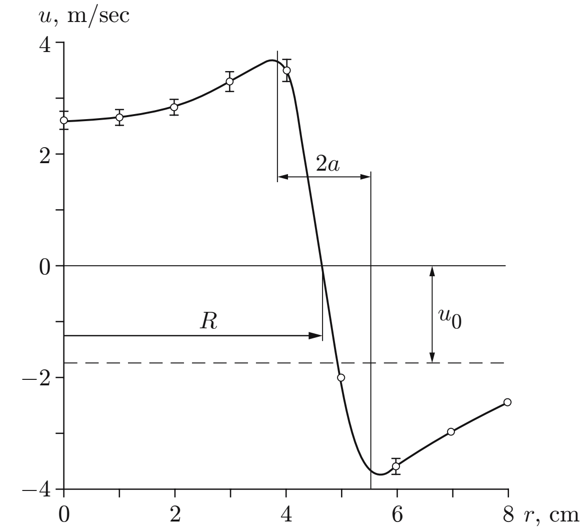

Fig. 2

Fig. 1. Pattern of the vortex ring motion.

Fig. 2. Velocity distribution in the vortex-ring plane $z = 0$.

moment when the velocity field was measured), which shows that the translational velocity of the examined vortex ring is $u_0 = (1.75 \pm 0.05)$ m/sec. The global distribution of velocities of this vortex ring is given in [5].

The velocity distribution in the vortex-ring plane ($z = 0$) in the vortex-fitted cylindrical coordinate system (z, r) is shown in Fig. 2. The distance along the r axis from the origin to the point of intersection of the curve $u(0, r)$ with the abscissa axis determines the radius of the vortex ring $R = 46.5$ mm. The velocity distribution in the neighborhood of the point $r = R$ has an almost linear character. The linear segment of the curve corresponds to the vortex core, and the distance $2a$ along the r axis between the extreme points at the ends of the linear segment of the curve can be taken as the vortex-core diameter. The dashed straight line in Fig. 2 parallel to the r axis corresponds to the translational velocity of the vortex ring u_0. The Reynolds number of the vortex ring, determined on the basis of the ring radius and velocity for a kinematic viscosity $\nu = 1.49 \cdot 10^{-5}$ m^2/sec, is $\mathrm{Re} = u_0 R/\nu = 4.54 \cdot 10^3$. It follows from the experimental results that the oscillograms of the vortex-ring velocity are smooth curves, while the distribution of smoke visualizing the flow has a layered character, i.e., the vortex ring is laminar.

Based on the velocity field, we found the stream function ψ and constructed the pattern of streamlines with a step $\Delta\psi = 4 \cdot 10^{-4}$ m^3/sec (Fig. 3). It is seen that the zero streamline ($\psi = 0$) consisting of the z axis and a certain closed curve divides the whole flow domain into two subdomains. For $\psi < 0$, the streamlines are closed, and the fluid circulates around the point $z = 0$, $r = R$. For $\psi > 0$, the streamlines are non-closed curves corresponding to a uniform non-separated flow around a solid bounded by the surface $\psi = 0$ with a velocity equal to the translational velocity of the vortex ring u_0. As the value of $|\psi|$ increases, the shape of the closed streamlines approaches the shape of a circle with the center at the point $z = 0$, $r = R$. The closed streamline $\psi = 0$ separates segments on the z axis, whose lengths differ within 10%. Vortex asymmetry in the direction of motion along the z axis can be attributed to the influence of medium viscosity. The shape of the surface of revolution formed by the closed streamline $\psi = 0$ is close to the shape of a flattened ellipsoid of revolution with a semiaxes ratio $h/l \approx 1.5$. The fluid bounded by the surface $\psi = 0$ is the atmosphere of the vortex ring and moves together with the latter. The volume of the vortex atmosphere is $V \approx 0.846 \cdot 10^{-3}$ m^3, and the coefficient of virtual mass [6] of the atmosphere in the direction of its motion is $\mu_z \approx 0.83$. The distributions of vorticity ω in two cross sections of the vortex, which were found on the basis of the velocity field, are plotted in Fig. 4, which shows that the vorticity is bounded by a bell-shaped curve with an amplitude $\omega_{\max} \approx 1600$ sec^{-1} in the neighborhood of the point $z = 0$, $r = R$.

Based on the experimental data [5], one can find the energy characteristics of the vortex ring. The energy of the vortex ring is the sum of the energy E_a of the fluid in the vortex atmosphere and the energy E_f of the fluid

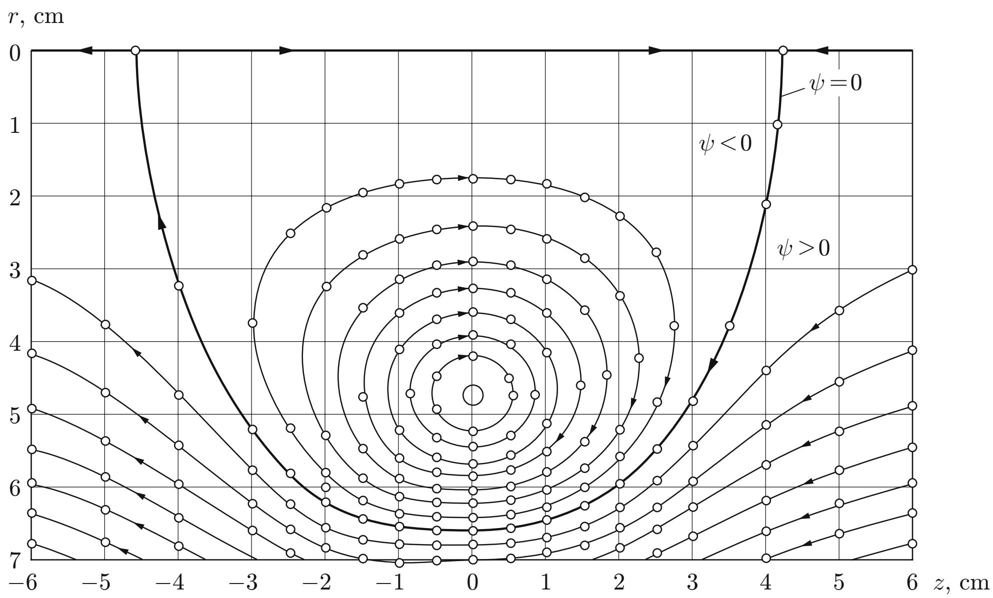

Fig. 3. Pattern of streamlines.

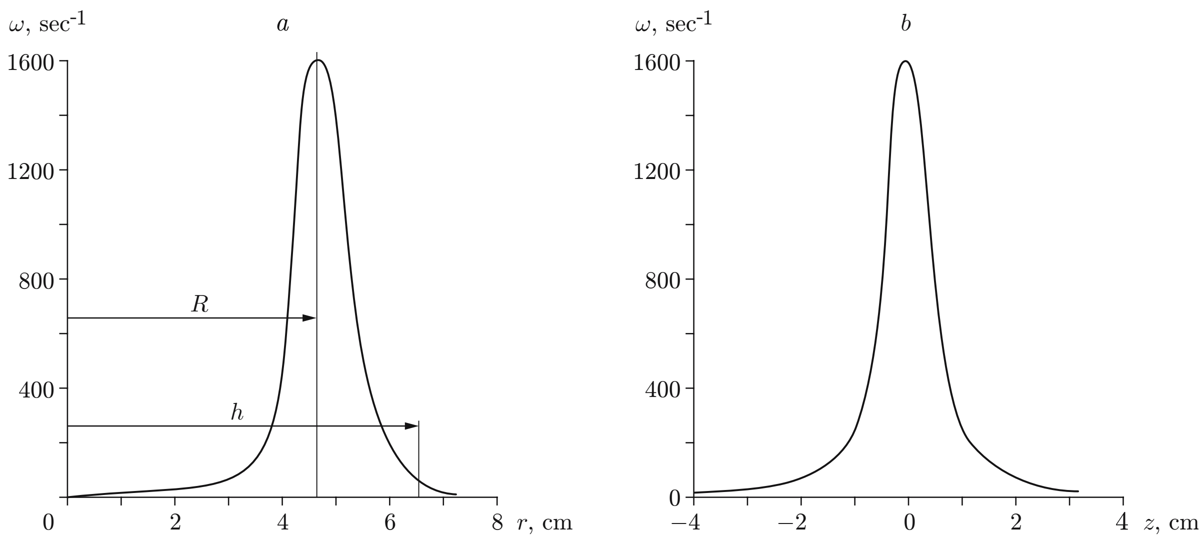

Fig. 4. Distribution of vorticity for $z = 0$ (a) and $r = R$ (b).

outside the vortex atmosphere. These components are determined separately. The energy of the vortex atmosphere is determined by direct integration of the velocity distribution over the volume V:

$$E_a = \rho \int\limits_V \frac{(u + u_0)^2 + v^2}{2}\, dV = 4.54 \cdot 10^{-3} \text{ J}$$

($\rho = 1.21$ kg/m^3 is the density of the medium and u and v are the axial and radial components of velocity, respectively). The energy of the fluid outside the vortex atmosphere is estimated on the basis of the coefficient of virtual mass μ_z of the vortex atmosphere: $E_f = \mu_z \rho u_0^2 V/2 = 1.3 \cdot 10^{-3}$ J. Thus, the total energy of the vortex ring is $E = E_a + E_f = 5.84 \cdot 10^{-3}$ J. It should be noted that $E_f/E_a \approx 0.22$, i.e., the kinetic energy of the flow outside the vortex-ring atmosphere is only 1/5 of the kinetic energy of the fluid in the vortex atmosphere. By comparing the energy of the vortex-ring atmosphere E_a and the energy $E_t = \rho u_0^2 V/2 = 1.55 \cdot 10^{-3}$ J of a solid of the same mass

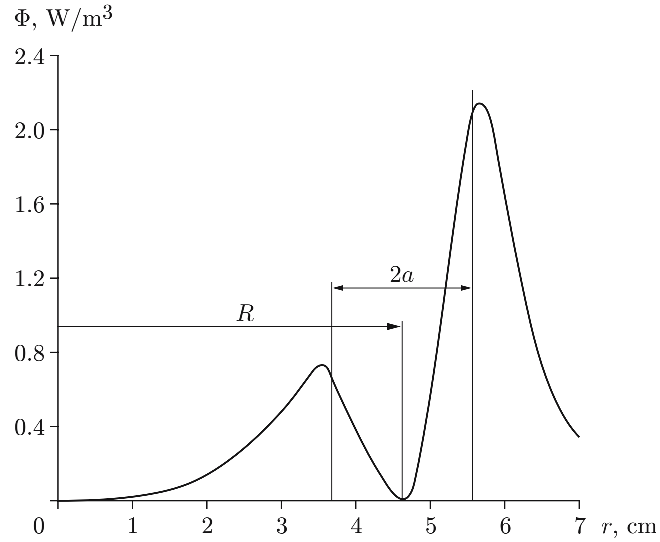

Fig. 5. Distribution of energy dissipation along the r axis in the plane $z = 0$.

moving in an empty space with a velocity u_0, we obtain $E_a/E_t \approx 2.93$, i.e., the energy of the fluid in the vortex atmosphere is approximately three times the energy of the corresponding solid.

Based on the distributions of velocity $\boldsymbol{u}$ and vorticity ω, we can find the energy dissipation $F = \partial E/\partial t$ of the vortex ring. The viscous dissipation of energy in a unit volume of the fluid in an axisymmetric flow is described by the expression

$$\Phi = 2\mu\Big[\Big(\frac{\partial v}{\partial r}\Big)^2 + \Big(\frac{\partial u}{\partial z}\Big)^2 + \frac{1}{2}\Big(\frac{\partial v}{\partial z} + \frac{\partial u}{\partial r}\Big)^2\Big],$$

where μ is the dynamic viscosity [4, 7]. The distribution of energy dissipation Φ in the vortex ring along the r axis in the plane $z = 0$ is plotted in Fig. 5. As it could be expected, the energy dissipation reaches the maximum value in the zone of high gradients of velocity near the vortex-core boundary and the minimum value inside the core, where the fluid motion is close to rotation of a solid. The energy dissipation in the fluid volume V is determined by the formula [4, 7]

$$F = \frac{\partial E}{\partial t} = \mu\Big[\iiint_V \omega^2\, dV + \iint_\Sigma \frac{\partial |\boldsymbol{u}|^2}{\partial n}\, d\Sigma - 2\iint_\Sigma [\boldsymbol{u} \times \boldsymbol{\omega}] \cdot \boldsymbol{n}\, d\Sigma\Big],$$

where V is the fluid volume considered, Σ is the area of the surface bounding the volume V, and $\boldsymbol{n}$ is the outward normal to the surface Σ. The energy dissipation is calculated separately inside the vortex atmosphere and in the fluid surrounding the vortex atmosphere. The calculations by the above-mentioned formula with $\mu = 17.9 \times 10^{-6}$ kg/(m·sec) show that the energy dissipation in the vortex atmosphere is $F_a = 0.82 \cdot 10^{-3}$ W. The energy dissipation F_f outside the vortex atmosphere is determined in a similar manner with allowance for vanishing of the volume integral over V, because we have $\omega \approx 0$ outside the vortex atmosphere. The calculations yield $F_f = 0.204 \cdot 10^{-3}$ W. Thus, the total energy dissipation of the vortex ring is $F = F_a + F_f = 1.02 \cdot 10^{-3}$ W. Based on this value of F, we can estimate the energy loss δE during the time $\delta t = 2R/u_0$ when the vortex ring moves to a distance equal to its diameter. As $\delta t = 0.054$ sec and $F = \partial E/\partial t$, then $\delta E = F\,\delta t \approx 0.055 \cdot 10^{-3}$ J; hence, $\delta E/E \approx 0.01$, i.e., the vortex ring loses approximately 1% of its energy during the time δt.

Let us compare the energy dissipation of the vortex ring with the loss of energy of a streamlined solid moving in a viscous medium. Numerous experiments show that the minimum drag is observed in non-separated flows around axisymmetric spindle-shaped solids with aspect ratios (ratio of the body length l to its transverse size d) $k = l/d =$ 5–6 [8]. As the drag of the body in a non-separated flow is determined by skin friction only, it is necessary to know the area of the body surface. The area of the surface of a streamlined body can be approximately estimated as the area of the surface of an extended ellipsoid of revolution with a semiaxes ratio $k = 6$ and with a volume equal to the volume V of the vortex-ring atmosphere. The length of such an ellipsoid is $L = (6k^2/\pi)^{1/3}V^{1/3}$, and the area of its surface is $S = 9\pi V^{2/3}/(2k^2 A)$. Here $A = 1 + (k^2/\sqrt{k^2-1})\arcsin(\sqrt{k^2-1}/k)$. Substituting the values $V = 0.846 \cdot 10^{-3}$ m^3 and $k = 6$, we obtain $L \approx 0.387$ m and $S \approx 0.0625$ m^2. The Reynolds number for this body moving with the vortex-ring velocity $u_0 \approx 1.75$ m/sec is $4.52 \cdot 10^4$. The hydrodynamic drag Q of the body can be determined as the drag of a flat plate with a wetted surface area S. Thus, $Q = c_f(\rho u_0^2/2)S$, where c_f is the skin friction coefficient of a flat plate wetted on one side. As the flow is laminar for $\mathrm{Re} < 5 \cdot 10^5$–$10^6$ [8, 9], the value of c_f can be determined by the Blasius formula: $c_f = 1.328/\sqrt{\mathrm{Re}} = 0.625 \cdot 10^{-2}$ (see [9]). The drag force of the body is $Q \approx 0.723 \cdot 10^{-3}$ N, and the power W necessary for this body to move is $W = Qu_0 \approx 1.265 \cdot 10^{-3}$ W. A comparison of the power W and the energy dissipation of the vortex ring $F = 1.02 \cdot 10^{-3}$ W shows that the energy spend during the vortex-ring motion is only slightly lower than the corresponding value for a streamlined solid of an identical volume. It should be noted that the energy loss during the actual motion of a solid can be somewhat higher than the value obtained here, because the motion of all solids at these Reynolds numbers is accompanied by flow separation and by formation of a cocurrent wake behind the body, which increases the drag force. The ratio of W to the energy dissipation F_f outside the vortex atmosphere is $W/F_f \approx 6.2$, i.e., the energy loss in the case of motion of a body similar in shape and volume to the vortex atmosphere with a corresponding moving boundary is much lower than the power necessary for a streamlined solid to move. Clearly, this gain in energy expenses can be utilized in practice only for a body similar to the vortex-ring atmosphere with a corresponding moving boundary, and creating such a body is a difficult engineering problem.

Based on the energy estimates obtained, we can try to give some qualitative explanations for the experimental results described above. Obviously, two factors are responsible for the greater path of the vortex ring, as compared with the path covered by the solid. First, the vortex-ring atmosphere is a body with a moving boundary, which ensures a non-separated flow with a lower drag force. Second, the total energy of the vortex ring consists not only of the energy of translational motion, but also of the energy of circulation motion of the fluid in the vortex atmosphere. As is shown above, the energy of the fluid in the vortex-ring atmosphere is approximately three times the energy of the solid of the same mass. Obviously, the mobility of the vortex-atmosphere surface is ensured by the circulation motion of the fluid in the vortex atmosphere. These two factors (higher value of the initial energy of the vortex ring and mobility of the surface bounding the vortex atmosphere) offer a qualitative explanation for the fact that the vortex ring covers a greater distance than a solid body of an identical mass, which is particularly true for a spherical body (balloon) with flow separation.

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